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Relational Interpretation of the Wave Function and a Possible Way Around Bell's Theorem

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The famous "spooky action at a distance" in the EPR-scenario is shown to be a local interaction, once entanglement is interpreted as a kind of "nearest neighbor" relation among quantum systems. Furthermore, the wave function itself is interpreted as encoding the "nearest neighbor" relations between a quantum system and spatial points. This interpretation becomes natural, if we view space and distance in terms of relations among spatial points. Therefore, "position" becomes a purely relational concept. This relational picture leads to a new perspective onto the quantum mechanical formalism, where many of the "weird" aspects, like the particle-wave duality, the non-locality of entanglement, or the "mystery" of the double-slit experiment, disappear. Furthermore, this picture circumvents the restrictions set by Bell's inequalities, i.e., a possible (realistic) hidden variable theory based on these concepts can be local and at the same time reproduce the results of quantum mechanics.

KEY WORDS: relational space; relational interpretation of the wave function; locality; Bell's theorem.

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1. INTRODUCTION

For many people, our quantum world still encompasses a few "mysteries":

- How can an object behave like a pointlike particle in some cases and like an extended wave in other cases? (Particle-wave duality)
- In particular, how can an object (say, an elementary particle) appear pointlike whenever it is measured directly, but on the other hand appear to be at two (or several) places at the same time when it is not observed (like

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the electron in the double slit experiment or the photon in a Mach-Zehnder interferometer)?
- How can the results of measurements on two entangled particles be correlated even when they are "miles away", although, according to the formalism of quantum mechanics, we are not allowed to assume that the output of these measurements is predetermined in adavance (Einstein's spooky action at a distance).

Furthermore, for all those, who favour an "objective and realistic" interpretation of the fundamental principles of our world, Bell's inequalities imply a major drawback. The experimental evidence in favor of quantum mechanics and the violation of Bell's inequalities in our physical world is overwhelming and no longer a matter of serious debates. The generally accepted conclusion is that the non-deterministic aspects of quantum mechanics are fundamental, i.e., already the assumption of a "hidden variable" determining the output of certain experiments in advance leads to contradictions unless we give up locality. Hence, all reformulations of quantum mechanics based on realism, like e.g. Bohm's quantum mechanics, include non-local interactions. The apparent discrepancy with the theory of relativity can only be overcome by proving that these non-local (hidden) interactions cannot be used for information transfer and should, therefore, not be interpreted as signals. However, the "spooky action at a distance", as Einstein called it, remains.

At a closer inspection, all the problems mentioned above are in some way or another related to what we mean by "locality", "position", "distance", "place", and in particular "to be somewhere". All definitions of locality (including the precise definition provided by the algebraic formalism of quantum mechanics) are based on a classical concept of space-time and may, therefore, not be completely consistent. "Position" refers to the points of a fixed background space. On the other hand, we expect that the concept of such a background space emerges as the classical limit of an underlying quantum theory of space and time. So, our present formulation of quantum theory is of a somewhat hybrid nature in that it describes quantum objects as being "embedded in" or "living on" a classical space. Although the interactions among the quantum objects are treated in a quantum mechanical way, the concepts of space and space-time and, in particular, the relations between quantum objects on the one side and space-time on the other side are footed on a classical description.

One might object by pointing out that Planck's scale is about 25 orders of magnitude away from atomic scales and, therefore, should not play any role in the quantum world as we see it today. However, there is at least one other structure of space (and space-time) which survives the 25 orders of magnitude: the metric field in special and general relativity. Space-time is more than a simple set of events, otherwise it would be impossible to measure and compare distances at different space-time points. This structure is usually taken for granted, but it

should be viewed as the large scale remnant of some unknown underlying structure of quantum space-time.

In this article, I will argue that already a simple reformulation of spatial concepts in terms of a relational interpretation might give us a new understanding of "locality". This concept does not only overcome the seeming discrepancy between realism and the violation of Bell's inequalities in our quantum world, but it almost trivially explains the other weird aspects mentioned above, like the particle-wave duality, the double-slit experiment etc. Although these concepts should rather be formulated in terms of space-time events and not in terms of spatial points, the major part of this article refers to a "relational theory of space", and only at the end there will be a few remarks about a generalization towards a "relational theory of space-time".

The expression "relational quantum mechanics" already exists in the literature and refers to Rovelli's (and others) interpretation of quantum mechanics (Rovelli, 2004), according to which states or the results of measurements do not have an absolute meaning, but all these concepts of quantum mechanics are to be understood in relation to the state of an observing system. (This interpretation is close to Everett's "relative state" interpretation of quantum mechanics (Everett, 1957), although the two interpretations differ with respect to the "many world" aspect which deWitt later attached to Everett's interpretation (deWitt, 1970).)

In a way, the present approach can be considered as a refinement or extension of Rovelli's concept. In relational quantum mechanics the state of a system is not attributed to the system itself (like in ontic interpretations) or to the observing system (like in epistemic interpretations), but the state is rather attributed to the boundary (or "cut") between the quantum system and the rest of the world, and it contains the information or knowledge which the rest of the world in principle has about the quantum system due to past interactions and present correlations between these two systems. The "refinement" discussed in the present article consists in the observation that in a world with relational space structure, the "cut" could be placed between the particle and space, i.e., the wave function $\psi(x)$ describes the "information" which space has about the particle. In general relativity, space-time has its own degrees of freedom which interact with other objects from which it can be deprived, so why not deprive the objects from space-time and consider them as entities of their own? "Position" should not be treated as an embedding of a particle, but as an external property arrising from the relations between this particle and those objects which make up space-time.

In the next Section 2, I give a brief introduction to models of relational spaces and to the general ideas of how a relational picture for quantum objects and spatial points may influence our understanding of quantum mechanics. Even if some of the more specific ideas elaborated in the following sections should turn out to be wrong, the general scenario explained in Section 2 may still be true. In Section 3, I will specify the ideas by making some assumptions about the type of

relations between quantum objects and spatial points. Section 4 describes a simple model for the propagation of relations and introduces a new concept of locality. The essential purpose of Sections 3 and 4 is to show that the relational picture fits well into the present formulation of quantum mechanics and that there is no need to change the formulas but to change the interpretations. In Section 5, I will raise the question whether all amplitudes in quantum mechanics express relations, and I will give some examples of relational concepts in todays standard model of elementary particles. Section 6 contains a few remarks about the extension of "particles in a relational space" to a model of "events in relational space-time". A brief summary concludes this article.

2. RELATIONAL SPACE

The clearest and most uncompromising formulation of a relational theory of space has been put forward by René Descartes in his "Principles of Philosophy," published in 1644 (Descartes, 1644). In the second part of his "Philosophiae," entitled "About the Principles of Corporal Things," Descartes argues that there is no such thing as empty space, and our whole concept of space is just an abstraction of what in reality are "relations" between bodies. For Descartes these relations express "immediate neighborship" or the "contact" of bodies. "Movement" is merely a change of these relations, i.e. a rearrangement of objects. The "bodies" may not always be visible to us, but "if God would remove from a vessel all the body" then "the sides of the vessel would thus come into proximity with each other."

Later, similar concepts have been put forward by Gottfried Wilhelm Leibniz. In his famous exchange of letters with Samuel Clarke (Clarke, 1716), he puts his concepts of a relational space and relational time against Newton's concepts of absolute space and absolute time. For Leibniz, space is just an abstraction of "the order of coexistences." Leibniz is less clear about the nature of the relations between physical objects and refers to them as "some relation of distance" (in a different context he speaks about "perceptions" (Leibniz, 1954)), but the fundamental ideas are similar to those of Descartes. Towards the end of the 19th century it was Ernst Mach who brought up the subject again (Mach, 1883). His work has greatly influenced Einstein in his development of the general theory of relativity.

More recently, the importance of a relational formulation of the fundamental laws of physics has been emphasized by Julian Barbour (amongst others), who, together with B. Bertotti, constructed a theory of mechanics based on relational principles (Barbour and Bertotti, 1977, 1982). In other approaches, the microscopic structure of space-time is modelled in terms of relational principles, like the causal sets of Sorkin (1991).

How does a relational space look like? Imagine certain objects which, for simplicity, will be represented by points, keeping in mind, however, that any notion of "extension in space" for an elementary object is meaningless in the relational view. Also for simplicity, I will distinguish objects representing "spatial points" and objects representing particles. In this section, I will consider only one type of relation. This relation is represented by a line. This leads to the concept of a graph. The relation is either present or not present. Mathematically, this can be expressed by the adjacency matrix:

$$A(x, y) = \begin{cases} 1 & \text{if } x \text{ and } y \text{ are related} \\ 0 & \text{otherwise} \end{cases} . \tag{1}$$

In the next section, I will add some hypotheses about the nature of the relations, and the concept of an adjacency matrix will be generalized.

The spatial points are related in such a way that, on large scales, space looks like a three dimensional manifold. A three dimensional lattice has this property, however, there is no reason (and no necessity) to assume that space at the fundamental level resembles a regular lattice.

The distance between two points is defined purely intrinsically as a suitable average of the lengths of paths connecting the points. The length of a path is given by the number of lines of this path. It will be important to notice that the (macroscopic) distance between points is not determined by the length of the shortest path alone, but by a suitable average over all paths (where a possible weighting of paths might depend on the length of a path). I will say more about "distance" later.

The "position" of an object is defined by its relation to spatial points. Therefore, the position of an object can be localized or extended, depending on whether this object has only relations to points which are close to each other or far apart from each other with respect to the intrinsic distance. It is even conceivable that a single object can be at two "places" simultaneously (see Fig. 2a), so the relational picture overcomes "the only mystery" (this expression is due to Feynman (1965)) of quantum mechanics. In this picture, a particle can be related to two places at the same time which may allow for a natural explanation of the double slit experiment.

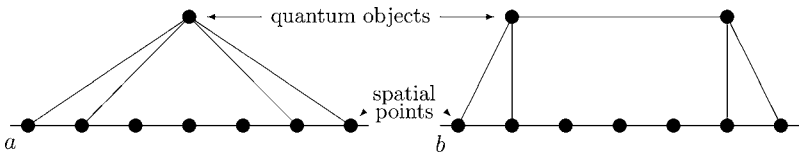

Fig. 1. (a) A relational particle can be at two places at the same time. (b) Two particles which are far apart with respect to the spatial distance can still be immediate neighbors.

Notice that only the relations matter, not the "length" of a line or the "position" of an object in a graphical representation.

Next, imagine two particle-like objects each having its relations to spatial points. With respect to the intrinsic relations among spatial points, these two particles could be far apart from each other. However, these particles may be immediate neighbors with respect to other relations (in the next Section, I will argue that measures of entanglement are a good candidate for the relations between particles, but for the moment I will not make any assumptions about the nature of these relations). With respect to this relation the particles are "immediate neighbors" (see Fig. 2b). Performing an experiment on one of the particles can immediately influence the other without violating locality, even though the particles look "miles apart" from each other with respect to their spatial relations. (In Section 4 the concept of locality is generalized to the dynamics of relations.)

In order that two objects are observed to have a large spatial distance even thought they are directly related to each other (and therefore there exists a path of length 1 connecting these objects), it is important that not only the shortest path on the graph contributes to the distance, but all paths. The fact that there exists a short-cut (via the relation between the particle-like objects) does not significantly change the macroscopic distance between two spatial points, which is obtained by a suitable averaging procedure. A huge number of such short-cuts, stemming, e.g., from many particles, could alter the macroscopically observable spatial distance between spatial points. (This observation might even be a starting point for bringing in general relativity. That the presence of matter changes the metric is one more feature which looks more natural in a relational theory of space. However, further conclusions are still very speculative.)

Thus, we have seen that the relational picture solves two of the major mysteries of quantum mechanics almost trivially: the fact that a particle can "be in" (in the sense of "have relations to") more than one place at the same time, and the "spooky action at a distance" in the EPR-scenario.

There is a third aspect of quantum mechanics which appears more natural in the relational picture. When the spatial relations of two identical objects are

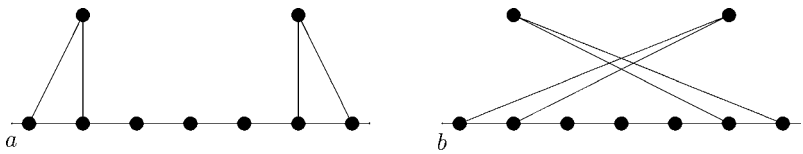

Fig. 2. (a) Two identical particles having relations to different spatial points. (b) The exchange of the two particles leads to the identical set of relations.

exchanged (see Fig. 2), the set of relations remains unchanged, i.e., the two situations not only look identical but they are identical. This is how they are treated in quantum mechanics.

3. THE WAVEFUNCTION AS THE "ADJACENCY MATRIX" OF RELATIONS

In this section, I will make the relational picture more concrete by adding a few hypotheses about the nature of the relations. This will lead to the simplified picture which is sketched in Fig. 3.

According to this picture, our present formulation of quantum mechanics includes essentially three types of relations, although we are usually aware of only one. The first (and well-known) relation emerges in the description of the interaction between quantum systems like elementary particles. Such interactions, in the context of quantum field theory attributed to the exchange of other particles, lead to entanglement which can be viewed as a quantum relation among quantum systems. Hence, quantum mechanics describes the relations among quantum objects. This aspect of quantum mechanics is not put into question. What is new, however, is the idea to consider entanglement as a "nearest neighbor" relation. Whenever two particles or two systems are entangled, they are, in a sense to be defined, immediate neighbors.

One might object that entanglement is not a binary relation. The information about an arbitrary entangled state of more than two particles cannot be decomposed into pairwise relations and thus cannot be represented by a simple graph. This is true and might hint at the possibility that the relational picture presented in this article has to be generalized from simple graphs to, for instance, abstract simplicial structures, including also ternary and higher order relations. Despite very intensive research, a satisfying theory for measures of entanglement is still missing (see e.g. Horodecki et al., 1996; Hill and Wootters, 1997; Wootters, 1998; Bruß, 2001, and

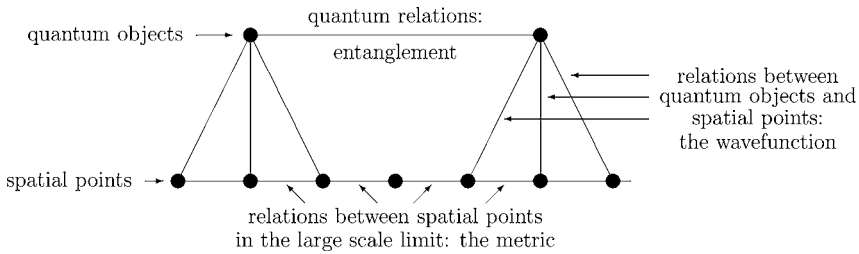

Fig. 3. Three types of relations are present in our formulation of quantum mechanics. Relations among quantum objects are due to entanglement, relations among spatial points should in a large scale limit lead to the metric, and relations between quantum objects and spatial points are described by the wave function.

references therein.) A characterization by graphs has been attempted in Plesch and Bužek (2003), and, more recently, Bob Coecke found a formulation in which entanglement can be characterized as a generalized relation in a categorial sense (Coecke, 2000). A more detailed theory of entanglement might influence the details of the framework presented here, but the general ideas remain untouched.

The second type of relations is usually not mentioned explicitly, but it is inherent in the description. These are the relations among spatial points (or space-time points, i.e. events). As we are not yet in the possession of a fundamental quantum theory of space and time, we do not know the nature of these relations on Planck's scale. Possible candidates are Rovelli's and Smolin's "loop space" (Rovelli and Smolin, 1990) or an extension of the spin networks of Penrose (Penrose, 1972). Whatever the theory at Planck's scale may look like, in a large scale limit these relations should lead to the emergence of distance and to the metric field $g_{\mu\nu}$ of general relativity. In lack of the fundamental theory, I will adopt here the simple relational model of the previous section: Spatial points are represented by the vertices of a network (or graph) and the relations between these spatial points will be represented by the lines of the graph. This model is to be understood as a "semi-phenomenological" description of space at small distances.

The third type of relations is usually not even mentioned implicitly, but may be the most important one for our understanding of quantum mechanics. These relations describe the connections between quantum objects (particles) and space (or space-time). On a fundamental level (at Planck's scale), there may be a complicated mixture of entanglement involving spatial points and quantum particles. In the large scale limit, which governs the phenomena of elementary particles and atoms, this mixture is effectively described by the wave function of quantum mechanics. So, rather than representing a strict "either-or-not" -relation, the lines in the figures presented above represent complex numbers, and these complex numbers express a semi-classical limit of the entanglement between spatial points and quantum objects. In this way the interference of relations can be explained, i.e., the fact that relations can "annihilate" each other by superposition. (On a more fundamental level such an annihilation can also be explained by opposite "flows" along the lines representing the relations. One may even speculate that the relations represent a "flux of (quantum) information".)

Let $\psi_e(x)$ be the wavefunction of an electron. We define the generalized adjacency matrix attributed to a system consisting of spatial points and an electron by

$$A(e, x) = \psi_e(x), \tag{2}$$

where e refers to the "object" electron and x to a labeling of the spatial points (compare Fig. 3). Thus one arrives at the following form of the generalized "adjacency

matrix" including spatial points x_i and quantum objects e_i:

$$\mathrm{Ad} \simeq \begin{pmatrix} A(e_i, e_j) \simeq \begin{pmatrix} \text{measure for} \\ \text{entanglement} \end{pmatrix} & A(e_i, x_j) \simeq \Psi_i(x_j) \ \text{ wave function} \\ A(e_i, x_j)^T \simeq \Psi_i(x_j)^* & A(x_i, x_j) \longrightarrow g_{\mu\nu} \\ & \text{(large scale limit)} \end{pmatrix} \tag{3}$$

4. THE DYNAMICS OF RELATIONS

In this section, I will speculate about the possible dynamics of the model introduced in the last section. In particular, I will propose a new locality principle, based on the dynamics of relations. At this point the notion of time enters. In lack of a fundamental theory, I will use a "semi-phenomenological" ansatz and consider "time" as a sequence of discretized steps and formulate an iterative equation for the entries of the adjacency matrix in Eq. (2). The objects themselves don't move. Movement is interpreted as a change of relations.

For the "propagation of relations" it seems close at hand to require the following general "locality principle": Two quantum objects e_1 and e_3 can only become related, if there exists a third quantum object e_2 such that e_1 and e_2 as well as e_2 and e_3 are already related. As this step-wise propagation of a relation involves a single "time-step" (which is expected to be of the order of the Planck time), it would appear to be "immediate" compared to macroscopic scales.

As far as entanglement among quantum systems is concerned, this locality principle can be demonstrated in a well-known example: the measurement on an EPR-state. Consider a system consisting of three subsystems: two entangled electrons e_1 and e_2 in an EPR-state, and a measurement apparatus (assumed to be in the state $|0\rangle_3$ initially). Hence, the initial state is:

$$|\Psi\rangle_{\mathrm{i}} = \frac{1}{\sqrt{2}}(|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2)|0\rangle_3. \tag{4}$$

The apparatus now performs a measurement on electron 2 (by an ordinary local interaction involving a flow of energy). This leads to entanglement between the apparatus and electron 2. If electron 2 were not entangled but in the pure state $\frac{1}{\sqrt{2}}(|\uparrow\rangle_2 - |\downarrow\rangle_2)$, this process could be described by:

$$\frac{1}{\sqrt{2}}(|\uparrow\rangle_2 - |\downarrow\rangle_2)|0\rangle_3 \longrightarrow \frac{1}{\sqrt{2}}(|\uparrow\rangle_2|+\rangle_3 - |\downarrow\rangle_2|-\rangle_3). \tag{5}$$

As an immediate consequence of this interaction, the already existing entanglement between electron 1 and electron 2 "swapps over" to the measurement apparatus

and the final state reads:

$$|\Psi\rangle_{\mathrm{f}} = \frac{1}{\sqrt{2}}(|\uparrow\rangle_1|\downarrow\rangle_2|-\rangle_3 - |\downarrow\rangle_1|\uparrow\rangle_2|+\rangle_3), \tag{6}$$

expressing entanglement of all three subsystems.

As a side-remark I should like to notice that the relational picture in its present form does not solve the measurement problem, i.e., it does not describe or explain the "collapse" of the state 6:

$$|\Psi\rangle_{\mathrm{f}} = \frac{1}{\sqrt{2}}(|\uparrow\rangle_1|\downarrow\rangle_2|-\rangle_3 - |\downarrow\rangle_1|\uparrow\rangle_2|+\rangle_3) \longrightarrow \begin{cases} |\uparrow\rangle_1|\downarrow\rangle_2|-\rangle_3 \\ \quad \text{or} \\ |\downarrow\rangle_1|\uparrow\rangle_2|+\rangle_3 \end{cases} \tag{7}$$

It is this collapse, not the interaction between electron 2 and the measurement apparatus, which destroys the entanglement and leads to a (non-entangled) product state.

As mentioned before, there is a slight difference between the relational picture of the propagation of relations and the standard quantum mechanical description: In the standard interpretation, the entanglement between the measurement apparatus and electron 1 evolves parallel to the entanglement between the apparatus and electron 2. In the relational picture presented so far, the entanglement between the apparatus and electron 1 occurs "one time-step after" the entanglement between the apparatus and electron 2 has been established. Apart from the fact that the build-up of entanglement is not an instantaneous "jump" but rather a continuous process initiated by the interaction, this difference of "one time-step" (which is assumed to be of the order of Planck's time, i.e., 10^{-44} s) cannot be directly observable. However, we do observe this difference indirectly as the limit on the propagation velocity (the speed of light) of objects. It would be interesting and at the same time a first test of this relational picture, if this finite propatation velocity for entanglement could be observed more directly, e.g. in condensed matter systems or other many particle systems.

In order to be more explicit about the dynamics of the other relations (the wave function and the spatial part of the generalized adjacency matrix), we need the adjacency matrix for the spatial points, i.e., we need to know $A(x, y)$. A simplifying assumption treats the spatial relations as fixed, i.e., space points do not take part in the evolution of the system and the matrix $A(x, y)$ does not change. In a more fundamental model, this will no longer be true: the relations among spatial points will fluctuate and change; however, the large scale limit—the metric—will remain constant. This puts severe restrictions on the fundamental dynamics of relations among spatial points.

Let $A(x, y)$ be the adjacency matrix of the graph (Eq. (1)), then we can define the so-called graph-Laplacian:

$$\Delta(x, y) = A(x, y) - V(x, y), \tag{8}$$

where

$$V(x, y) = \left(\sum_z A(x, z) \right) \delta(x, y) \tag{9}$$

is the diagonal valence matrix of the graph ($\delta(x, y)$ being the discrete Kronecker-Delta). $\Delta(x, y)$ is the discretized analogue of the standard Laplacian on a manifold. The discretized (free) Schrödinger equation for the relations $A(e, x; t)$ of particle e now reads:

$$\mathrm{i}A(e, x; t + 1) = \mathrm{i}A(e, x; t) - \mu \sum_y \Delta(x, y)\, A(e, y; t), \tag{10}$$

where μ is some constant which in the continuum limit has to be renormalized to the inverse of the mass m of the particle. This may be considered as a discretized version of quantum mechanics, and in the continuum limit the wavefunction obeys Schrödinger's equation.

Employing this formalism, we also arrive at an interesting interpretation of Feynman's "summation over paths." In quantum mechanics, the general solution of Schrödinger's equation can be written in the form:

$$\psi(x, t) = \int dy\, K(x, y; t)\, \psi(y, 0). \tag{11}$$

Formally, the kernel $K(x, y; t)$ can be represented as a sum over all paths of "length" t from point y to point x, where each path is "weighted" by a phase depending on its classical action. Feynman's "summation over paths", or, more generally, "summation over histories" is an intuitive expression. The formalism requires, however, that "a particle propagates along path 1 AND path 2 AND path 3 ...", which is difficult to understand. If we interprete the wave function as encoding the relations between a particle and space, Feynman's integral reads "propagation of relation 1 along path 1 AND relation 2 along path 2 AND ...." (According to the locality principle, relations can also split or merge; this finally leads to the sum over "all" paths.) Interpreted in this way, the sum over histories appears much less "mysterious."

5. ARE ALL "PROPERTIES" RELATIONAL?

In standard quantum mechanics we are free to choose a basis in Hilbert space. If we denote by $|\psi\rangle$ some vector in this Hilbert space representing the state of the system, Schrödinger's wave function is usually expressed in the position

base: $\psi(x) = \langle x|\psi\rangle$. However, we can also choose the momentum base instead ($\tilde{\psi}(p) = \langle p|\psi\rangle$) or the (orthonormal) base defined by the eigenvectors of any other observable.

In the relational picture outlined in the previous sections, the position basis is distinguished. We have assumed that the "spatial points" have an objective reality in the same sense as the particles have an objective reality. The relation between these two objects is expressed by $\psi_e(x) = \langle x|\psi\rangle$. The transformation to the momentum representation is obtained by a Fourier transformation like in standard quantum mechanics. However, it is not clear in what sense $\langle p|\psi\rangle$ can be interpreted as a relation between two "objects". In this picture there is no objective "momentum point", although formally one can represent the relational picture in any other basis. In an even more general setting one would like to gain back the lost symmetry and interprete the transition amplitudes $\langle a|\psi\rangle$ as a relation between an "object" ψ and an "object" a.

In its present formulation, the model presented here is far from this goal. However, I will show that even in standard quantum mechanics and quantum field theory, many "properties" of particles emerge through relations between this particle and its environment. The purpose of this section is to show that relational properties are nothing new even within standard theories in physics.

One of these properties is the mass m of a particle in the standard model of elementary particles. Usually one starts from a Lagrangian in which all particles are treated as massless (see, e.g., Weinberg, 2000). As a result of spontaneous symmetry breaking, the Higgs-field acquires a non-vanishing expectation value, and due to the Yukawa-type interaction between the other particles in the model and this background Higgs field the particles acquire a mass.

Yet, we do not have to employ the mechanism of symmetry breaking in order to find relational properties in quantum field theory. The observed mass m and charge e of the electron in quantum electrodynamics are not properties of the "bare" electron. In the standard interpretation, these properties are determined by the interactions between the electron and the quantum fluctuations of the environment (expressed essentially by the higher order corrections of the two- and four-point functions in Feynman's perturbation theory). In standard QED this leads to the renormalization of the mass and the charge of the electron.

As a third property, I should like to mention the formalism by which "spin" is usually described in quantum mechanics. The Hilbert space of square-integrable functions handles the spatial degrees of freedom of a particle, and in addition a two-dimensional complex vector space refers to the spin of the particle. This tensor space construction expresses the fact that the spin degrees of freedom of a particle are independent of the spatial degrees of freedom. The spin state and the spatial state of the particle can even be entangled: In the neutron interference experiments (Hasegawa et al., 2003), the intermediate state of the neutron can be

written as:

$$|\psi\rangle = \frac{1}{\sqrt{2}}(\psi_1(x)|\uparrow\rangle + \psi_2(x)|\downarrow\rangle), \tag{12}$$

where ψ_1 and ψ_2 refer to two different paths of the particle through the interferometer. A similar situation occurs when a photon hits a polarization beam splitter which reflects photons of one polarization and transmits photons of the orthogonal polarization. Thus, the formal description of spatial and spinorial degrees of freedom of a particle resemble the description of multi-particle systems.

6. RELATIONAL MODELS OF SPACE-TIME

In this final section, I should like to make some remarks about the generalization of the above described model of relations between particles and spatial points towards a theory of relations between events and space-time points. At present, these remarks are very speculative.

In 1990, Rudolf Haag raised a very fundamental question (Haag, 1990): What are elementary events? Despite the fundamental importance the concept of events has for general relativity, it has never been formalized within the context of quantum theory or quantum field theory. (A more recent update of his ideas can be found in Haag (2004).) We know that within the framework of perturbation theory, we can express the transition amplitudes of quantum field theory by a sum over Feynman integrals and, in a graphical notation, by a sum over Feynman graphs. These Feynman graphs show elementary events (like the emission or absorption of a photon by an electron in QED), but these events are not "factual", they have to be considered as "virtual" events or "possibilities". Haag studies the transition from possibilities to factuality by looking at increasing clusters of virtual events, finally arriving at partitions of our universe into factual subsystems.

In the Section 4, I have already explained that the "summation over histories" (including a summation over all possible kinds and locations of elementary events allowed by the classical action) appears much more natural in a relational interpretation. The "elementary events" do not involve actual particles propagating through space, but only certain relations between particles and spatial points. In this way there is no need to explain a transition from possibilities to facts, but all relations may be considered as factual. Strictly speaking, however, the "event" of, say, the scattering of two electrons is distributed all over space-time.

On a closer inspection, however, the generalization of the scenario described in the previous sections from "spatial points" to "space-time events" turns out to be more problematic. First, in order to reproduce the standard results of quantum field theory, a certain event can be related to all other space-time-events, independent of whether the two events are space-like or time-like (i.e., independent of whether they are causally related or not). This does not violate the principle of causality

as long as the relational weights reproduce the known causal Green functions (whose real parts are also non-zero for space-like events). In a large scale limit (where "large" can mean anything larger than Planck's scale) these relations can still reproduce other discretized models of space-time (like causal sets (Sorkin, 1991) or random networks (Requardt, 2000)).

While the relational picture can reproduce the contribution of a single Feynman graph (including the integrations over all internal space-time points of elementary events), the complete theory requires also a sum over all possible Feynman graphs, i.e. over all possible combinations of elementary events. In the present picture, this summation cannot be replaced by a single set of relations. The "superposition principle for a second-quantized theory" turns out to be more subtle and may require an even more general formalism of relations.

7. SUMMARY

It is argued that if we treat the position of a particle not as an embedding into some background space but as an expression of the relations between this particle and spatial points, and if we interprete the wave function in quantum mechanics as encoding these relations, we arrive at a relational interpretation of quantum mechanics which not only solves some of its "mysteries" (the particle-wave duality or the spooky action at a distance) but which might also be a way to circumvent the restrictions set by Bell's inequalities on "local realism" in a hidden variable theory. The aim of the present article was to show that in such a framework the formalism of quantum mechanics remains almost unchanged, but the interpretation of many expressions becomes more natural.

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